Improving Pre-service Teachers’ Understanding of Complexity of Mathematics Instructional Practice through Deliberate Practice: A Case Study on *Study of Teaching*

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Abstract

Many teacher education programs are criticized for their failure to prepare pre-service teachers to utilize and transfer their knowledge of content and educational principles into authentic classroom teaching. Studies also consistently report that pre-service teachers’ ability to identify what is noteworthy about a mathematics classroom situation was lacking. This study investigated the effect of the intervention program, *Study of Teaching*, on pre-service teachers’ acquisition of three designated mathematics teaching skills, how much they understood about the complexity of mathematics teaching, and their professional growth. We began with the assumption that pre-service teachers’ ability to discern the ‘critical aspects’ of mathematics classroom instruction could be enhanced through enacting reduced length microteaching sessions. Our results indicate that reduced microteaching in the setting, as in our intervention program, provide pre-service teachers with a safe environment to practise teaching. In general, this project found that pre-service teachers’ awareness of the complexity and intricacy of actual classroom teaching of mathematics had been greatly enhanced following this intervention.

Keywords: Mathematics teacher education, Study of Teaching, Micro-teaching

1. Introduction

For more than a decade there has been critique of the disconnection between theory and practice in teacher education programs. Many teacher education programs are criticized for their failure to prepare pre-service teachers to reconstruct their knowledge of content and educational principles into authentic classroom teaching (Eilam & Poyas, 2009; Goodlad, 1990; Korthagen & Kessels, 1999; Korthagen, Loughran & Russell, 2006; Lai & Lo-Fu, 2013). When designing a learning environment aimed at reconciling the dilemma of the theory-practice nexus in primary mathematics education programs, educators attempt to represent ‘real’ teaching practice in an authentic way to pre-service teachers (Oonk, Goffree & Verloop, 2003). In the construction of this environment, mathematics educators often include a classroom focused assessment task that requires pre-service teachers to work in small groups on designing lesson plans; conduct teaching in genuine classrooms in schools; analyse and evaluate their peers’ teaching; and finally give feedback to peers for improvement. Typically the goals of such assessment are, firstly, to experiment with teaching at a micro-level – teaching is scaled down in terms of instruction time, class size, and instructional strategies used (Sherin, 2003); and secondly, to develop pre-service teachers’ ability to analyse lessons.

Studies such as Alsawaie and Alghazo (2010), Santagata, Zannoni and Stigler (2007), Seago (2003), Star and Strickland (2008), and van Es and Sherin (2002) however, consistently report that pre-service teachers’ ability to address a range of issues is lacking. Issues such as: identify what is noteworthy about a mathematics classroom situation, to connect the specifics of mathematics classroom interactions with the broader principles of teaching and learning they represent, and to use what they know about the context to reason about mathematics classroom events (van Es & Sherin, 2002, p.573). For example, findings indicate that pre-service teachers are not particularly observant about issues of content and tend to view a lesson merely as a chronological but disconnected sequence of events (Star & Strickland, 2008). Other research findings (*inter alia* Eilam & Poyas, 2009) report that pre-service teachers rely heavily on the affective components of teaching, and tend to disregard the cognitive aspects of teaching episodes in which they are engaged. Researchers such as Eilam and Poyas (2006; 2009), Jaworski (2004), and Korthagen, Loughran and Russell (2006) conclude that pre-service teachers’ inattention to features of the classroom is due to their lack of awareness of the complexity and intricacy of actual classroom teaching of mathematics. Further, Eilam and Poyas (2006) advise that lack of awareness of complexity may impede teaching performance and therefore requires special training.

This paper will first discuss the complexity of mathematics teaching and the inherent difficulties in raising awareness of complexity. Second, a modelling approach in microteaching for the acquisition of some teaching skills and promoting pre-service teachers’ awareness of the complexity of mathematical instructional practice will be discussed. Next, an intervention program, *Study of Teaching,* which was conducted in a teacher education program for one semester and which used a modelling approach in microteaching will be introduced. Finally, the paper reports the results of this case study investigation into *Study of Teaching* (Reid, 2011).

2. The complexity of mathematics teaching

Classroom teaching is frequently presented as a highly dynamic, constantly changing process characterized by concurrent interactions between multiple factors such as "physical conditions, authority structures, attitudes, teacher-pupil relationships, text books, examinations, and time" (Jaworski, 2004, p.17). These aspects combine inconsistently across different classrooms and thus create different scenarios for teachers to cope with (Eilam & Poyas, 2006; Lampert, 2001). In many pre-service teacher education programs, because of the incoherence and inconsistency of the structure of the courses, what pre-service teachers learn about teaching in universities tends to be compartmentalised – that is, failing to take into account the complexity of a real teaching situation. Consequently, teaching techniques that pre-service teachers are taught do not support them to make knowledge connections for use in real and different situations. Therefore, a gap opens between what is taught in pre-service setting (i.e., theory) and 'real world' applications (Mason & Spence, 1999). Furthermore, Jaworski (2004) reminds us that, whilst many tertiary educators had been teachers in schools, their current ways of knowing teaching and school culture often lack "currency", as well as having been influenced by the university context and requirements, and thus often differs from teachers who currently teach mathematics in schools. Jaworski's argument captures differences between the thinking and conditions at university and those in schools. Thus, the reason for the dilemma of the theory-practice nexus in mathematics education could be because pre-service teachers do not realize that teaching is more complex and difficult than is presented by those who advise them on how to teach (Desforges & Cockburn, 1987).

There is however, a growing body of research aimed at developing pedagogical approaches for promoting pre-service teachers' practical wisdom (Lunenberg & Korthagen, 2009), knowledge of knowing-to act in the moment (Mason & Spence, 1999) and awareness of the characteristics of classroom complexity (Eilam & Poyas, 2006; 2009). This research has enabled pre-service teachers to study the complexity of teaching practice in some detail, and has assisted teacher educators to enhance practical aspects of teacher education programs. The main objectives of these approaches are to "advance pre-service teachers' ability to identify cognitive aspects of implicit, concurrent teaching-learning processes and their interrelationships" (Eilam & Poyas, 2006; p. 339) and to foster their "sensitivity for and awareness of the essentials of a particular practice situation that shape our perception of this situation, and help us find possible courses of action" (Lunenberg & Korthagen, 2009; p. 226). The existing literature provides sound knowledge and understanding of what skilful inside-the-classroom routines (Grossman & McDonald, 2008; Lampert & Graziani, 2009) can deliver in terms of production of effective teaching. Despite the value of these approaches, Grossman and McDonald (2008) argue that university teacher educators tend to leave the development of subject specific pedagogical skills in the interactive aspects of teaching almost entirely to field experience, rather than address them systematically in teacher education programs. Grossman and McDonald indicate that the part of what differentiates "experts" of teaching from novices is not only their ability to view a lesson's underlying structure but their ability to engage in "deliberate practice" (Ericsson, 2006). Likewise, Ball and Cohen (1999) argue that what teachers need to learn has to be learned in and from "practice" rather than in preparing to practice. Similarly, Lave (1996) proposes "teaching as learning in practice" (p.149) while Jaworski (2006) highlights teaching as learning in practice in the way that the process of professional growth continues throughout practice. Teaching, according to Ball and Cohen (1999), 'requires improvisation, conjecture, experimenting and assessing' (p.10). The subject specific clinical aspects of practice such as giving mathematical instructions, delivering mathematical reasoning, using manipulatives, probing students' mathematical thinking and giving feedback to students, all of which are mathematical instructional practices, are also critically important components of quality teaching, and as such, need to be made explicit within university teacher education programs. Thus, this paper proposes that pre-service teachers' understanding of the complexity of mathematics instructional practice can be improved through engaging in "deliberate practice". From this position, the question arises as to how and what to deliberately practise, so as to cognitively bring pre-service teachers' attention to the discernment of the complexity of mathematics instructional practice.

3. Use modelling approach in microteaching to investigate the complexity of mathematics instructional practice

It is widely accepted that learning is not an affair of telling and being told, but an active constructive process in which learners enact a newly taught task and then construct their own knowledge (Dewey, 1916). Building on this idea, this paper postulates that enactment of complex practice in settings of reduced complexity could in some ways provide a platform for pre-service teachers to practise teaching. Through enacting reduced length microteaching, pre-service teachers are exposed to the complexity of teaching and, therefore, are able to discern the critical aspects of mathematics classroom instruction. Willis and Cifuentes (2005; p. 61) suggest that for pre-service teachers to master teaching practice, their "instruction must be situated in an authentic context that resembles that of the classroom teacher to enrich their learning process by providing realistic experiences that more easily transfer". The approach used in this study provides such experiences.

The use of microteaching within teacher education dates back to the early 1960s. Microteaching is used in the instruction of pre-service teachers to assist them in their mastery of specific micro-skills, such as explaining a mathematical concept, using manipulatives and asking questions. Nowadays, its original focus has been expanded to introduce pre-service teachers to the complexity of teaching and to explicitly connect theory to practice (Pringle, Dawson & Adams, 2003). Higgins and Nicholl (2003) point out that microteaching provides pre-service teachers with opportunities to "take risks" with teaching in a safe environment (i.e. without fear of failure) so that their confidence and self-awareness can be developed. The positive impact of microteaching on teacher education is also characterized by the provision of feedback. Pre-service teachers can be provided with critical feedback on their performance, by their supervisors, instructors and peers straight after they have performed their teaching, allowing them to determine how they could improve their skills (Daniel, Auhl & Hastings, 2013). Further, research on modelling in microteaching (Griffiths, 1975; MacLeod, 1987; and Pegg, 1985) indicates that if cueing, coding or labelling of modelled activities is provided, then the effectiveness of modelling is enhanced (McDonald & Allen, 1967; Claus, 1968). Similarly, Pegg (1985) proposes that modelling with some form of discrimination training can magnify the effect - this "model" ultimately provides for and governs the ways in which pre-service teachers actually teach.

Building on these ideas, the study reported here used a modelling approach in microteaching. According to Jaworski, "teachers' learning is not accidental" so we chose to use specific teaching skills for pre-service teachers to practise with purposeful activities with clearly defined goals (2004; p.27). In the study, teaching practice was focused on practising three subject specific teaching skills: explaining a mathematical concept, using manipulatives, and sequencing a series of modelling tasks for scaffolding students' learning - this was a guiding framework for the intervention program, *Study of Teaching* (Reid, 2011). Overall, the study aimed at investigating

a. first, the effects of skill practice in the intervention program on the acquisition of three teaching skills, to what degree pre-service teachers understood the complexity of teaching in relation to the three teaching skills; and
b. second, what other "critical aspects" of mathematics classroom instruction pre-service teachers discerned after the intervention program.

The overarching research aim was to understand the effect of the intervention program, *Study of Teaching*, on pre-service teachers' awareness of the complexity of mathematics teaching.

4. Method

4.1 The intervention program – Study of Teaching

The *Study of Teaching* (Reid, 2011) is fundamentally a **modelling approach in microteaching** where students encounter core practices of teaching (Grossman & McDonald, 2008; Grossman, Hammerness & McDonald, 2009) in a safe learning environment. Pre-service teachers have the opportunity to watch "practice" being modelled by expert practitioners, and then they rehearsed and practised with and for a group composed of their peers, academic instructors and practising teacher mentors, receiving and providing feedback along the way. The pre-service teachers were encouraged and supported to deepen their thinking and develop their capacities to identify quality practice, safe from the gaze of the pressure of "assessment" by an associate teacher. In the study reported here, "practice" will be focused on three mathematics teaching skills: explaining a mathematical concept, using manipulatives, and sequencing a series of modelling tasks for scaffolding students' learning. The following section will give a brief guiding framework for practicing these three teaching skills.

The intervention program identified the three modes of representation (i.e., concrete, pictorial and symbolic) for explaining mathematical concepts as one of the teaching skills to be practised. Research (*inter alia* Berthold & Renkl, 2009; Cabahug, 2012) consistently reports positive impacts of multiple representations on students' learning. According to NCTM (2000), teachers' ability to use multiple representations for explaining mathematical concepts is essential "in supporting students' understanding of mathematical concepts and relationships" (p.67). Scholars such as Presmeg (1999), Pape and Tchoshanov (2001), and Yakimanskaya (1991) point out that the use of different modes of representation such as concrete objects and pictures leads to improvement of primary students' mathematical abilities and development of their advanced problem solving skills. Their argument aligns with Bruner's (1966) learning theory that, through early exploration of concrete materials and pictures, students become more competent in interpreting abstract symbolic representations of mathematical ideas and operations. Likewise, Bruner (1966) identifies three different modes of representation that an individual develops: enactive (i.e., concrete), iconic (i.e., picture) and symbolic. The concrete level of representation involves movement and manipulation of concrete objects. The pictorial level is associated with images in the mind's eye or actual pictures. The symbolic level deals with such symbols for ideas as words or mathematical symbols.

The intervention program also determined that it was worthwhile for pre-service teachers to practise operating manipulatives. Piaget and Inholder (1971) point out that children of ages 6 to12 usually think in terms of concrete situations rather than abstract ones. A child can do more abstract thinking when given physical objects to manipulate. If a child is not given direct experiences with objects and materials, it is unlikely that (s)he will develop symbolic operations such as doing algorithms that should come later. Piaget further states that children actively construct knowledge as they manipulate and explore their world, and that their cognitive development takes place in stages (Berk, 1997). Likewise, **s**cholars such as Levenson, Tirosh and Tsamir (2004), Levenson, Tsamir and Tirosh (2010), and Raman (2002) argue that primary school students may be too young for rigorous mathematical explanations, but can be convinced by operating semi-structured manipulatives and models. These researchers argue that this is so because, when teaching an abstract concept which is unfamiliar to students, concrete objects/manipulatives provide students with visual assistance to impose a mathematical relationship on the object, and to transit their understanding from a concrete level to an abstract level. Australian and Hong Kong teachers value the role of concrete objects in mathematics teaching because concrete objects offer greater opportunities for fostering mathematical understanding (Bryan, Wang, Perry, Wong & Cai, 2007).

Procedural variation, by varying the conditions of the original problem, provides a hierarchical system of experiencing processes through forming concepts (Gu, Huang & Marton, 2004). It aims to provide a process for formation of concepts stage by stage (see for example, Lai & Murray, 2012). The underlying theory of variation is that certain invariances characterize certain ways of experiencing and seeing a particular phenomenon (Marton & Booth, 1997). Thus, what is varied and what remains invariant are intended to have direct impact on students learning a particular concept. Watson and Mason (2005) argue that because some critical features of problems are invariant while others are changing, learners are able to see the general through the particular, to generalize, and to experience the particular. Likewise, Watson and Mason (2006) point out that "mathematics tasks that carefully display constrained *variation* are generally likely to result in progress in the ways that unstructured sets of tasks do not" (p.92). They further claim that a series of modelling tasks which embed *variation* create generalization and generalizations are the raw material for mathematical conceptualization. Building on this idea, in this study, sequencing a series of modelling tasks that embedded variation was considered a specific teaching skill that was worthwhile practising in the intervention program.

4.2 The research process and participants

The participants in this study were members of a cohort of second year primary teacher education students, who volunteered to participate after an open invitation was extended to all students in the course. All the volunteers had participated in the *Study of Teaching* program the previous year, where the curriculum focus had been reading, with a pedagogical attention to enacting this for different ages and texts. For the mathematics teaching as presented in this paper, twelve pre-service teachers accepted the invitation but only ten stayed active to the end, with one who was an infrequent attendee.

Participants were required to attend 4 hours of "instruction and practice" each week for 6 weeks and commit 4 hours of "team teaching" in a rural school each week for 3 weeks. In the early weeks of the program, two academics created videos, where they modelled core mathematics teaching practices, using local children as the “class”. These videos were then made available to the participants to watch and “learn” how to teach the basic concepts. The participants were able to “practise” the teaching techniques as discussed in the previous section and also to implement the language models used by the academics. Further, academic staff and the participants engaged in discussions focussing on language, strategies, use of manipulatives and importantly, sequencing of tasks. The sequencing/scaffolding enabled the participants to break each algorithm down into individual parts to assist them to develop their understanding of how children “learn”. The discussions required them to question their “beliefs” about the use of manipulatives and how they thought the fundamental mathematics skills should be taught. In this way, the aspects of instruction were explicitly taught to participants.

Each participant was then required to model the teaching, based on the input provided by the academics, to the rest of the group, including all members of the research team. Each presentation was video-recorded to enable the participants to view their own and each other’s presentation. They were uploaded to an “interactive learning space” which was only accessible to the participants. Each participant was provided with feedback at the time and given the opportunity to re-present their lesson. The modelling, explicit teaching and rehearsal of teaching practices is recognised as important by a number of researchers (Hiebert & Morris, 2012; Zeichner, 2012) and enacting this in the more protected environment (Willis & Cifuentes, 2005) of the mathematics lab allowed participants to make mistakes and revisit their interpretations of the modelling without any concern in relation to assessment.

After 6 weeks of such sessions, the team of participants and academics visited the rural school and classroom where the “teaching” was to occur. The initial visit enabled the participants to meet the classroom teacher and observe her teaching style, as well as meet and interact with students in the class. All participants had the opportunity to interact with the students during this time to ensure that they had some familiarity with each other prior to the teaching.

In self-selected groups of 4 to 5, the participants planned a mini-lesson using the skills they were developing in “practice”. The content was determined by the classroom teacher. On the second visit to the school, Group 1 taught half of the class and Group 2 observed this lesson, while the associate teacher taught the rest of the class in another room. After 25 minutes, the participant groups swapped roles and so did the children. Consequently, Group 2 had the benefit of watching Group 1 in action. At the conclusion of both sessions, the whole of the team sat together to provide feedback. The participants were requested to provide critique first, then the academics. This process was repeated the following week but the Groups changed the order of teaching.

The participants' teaching practice was videoed at every stage of the program, not as a data source for the researchers but rather as a reflective learning tool, so participants could observe themselves in action. This served to make the feedback more pertinent, as commentary could be easily matched to actions. At the conclusion of this project, the 10 participants attended a group interview.

4.3. The interview

The whole team of participants and academics attended a group interview a week after the completion of the intervention program. The academics took the role of interviewer and led the discussion. The interview took one hour and consisted of two parts. The whole process of interview was audio recorded.

In the first stage, the participants were invited to tell what they had learnt after practicing mathematics pedagogical micro-skills which included explaining a mathematical concept, using manipulatives, and sequencing a series of modelling tasks for scaffolding students' learning. This part of the interview investigated the first research aim: the effects of skill practice on the acquisition of the three teaching skills and how much they understood the complexity of teaching in relation to the three teaching skills.

For the second stage of the interview, interviewers probed participants to think about what cognitive aspects of teaching mathematics that they felt they understood better than those pre-service teachers who did not participate in the program and what features they would look for in determining an effective mathematics lesson. Participants were asked to identify 'critical aspects' of mathematics classroom instruction in an attempt to ascertain what participants understood about the complexity of mathematics instructional practice (i.e., other than the three teaching skills) after completing the intervention program. This part of interview investigated the second research aim: what other “critical aspects” of mathematics classroom instruction pre-service teachers discerned.

The interview was transcribed and the transcription was sent back to the participants and academics for member checking. The analysis of the interview episodes employed the concept of “reflection-on-action” (Schon, 1983) which was characterized by the participants’ posteriori analysis of their own practices. In such reflections, participants could systematically apply their acquired concepts of mathematics teaching, reasoning for mathematics instructional skills and analytical strategies to understanding and assessing their practices (Garcia, Sanchez & Escudero, 2006). In this study, reflection-on-action was a means for the researchers to approach the participants’ understanding of the complexity of mathematics instructional practice after the intervention program.

5. Results and discussion

Enacting complex mathematics instructional practices provides pre-service teachers with a platform for practising subject specific teaching skills and discerning ‘critical aspects’ of classroom instruction. This section describes the participants’ professional growth in their understanding and awareness of the complexity of mathematics teaching. The analysis of data and reporting of findings set out below is linked tightly to the two research aims as outlined in the earlier section. The argument is constructed on the responses of participants in describing how their beliefs about teaching mathematics had changed.

5.1 The effect of the three teaching skills practice on pre-service teachers’ understanding of the complexity of mathematics teaching

5.1.1 Explaining a concept

The responses from participants in this intervention program indicate that they had not initially recognised the need for using three modes of representation (i.e., concrete, pictorial and symbolic) to help students construct abstract concepts, due to their perception of mathematics teaching as teaching for numeracy and proficiency, both of which only require procedural knowledge (i.e., algorithms). The following notes demonstrate this point.

Participant 1: We're not used to teaching that way (i.e., use three modes of representation), and we're not used to, well we see [an algorithm] and we figure it out in our heads straight away.

Participant 9: It's hard to explain [the concept by using objects and pictures] but why we need to [explain in this way].

They appeared to not know what conceptual knowledge and mathematical reasoning were required to explain the legitimacy of a particular algorithm, such as subtraction of whole numbers. The participants recalled the challenge they faced when they were first asked to use concrete objects and pictures to explain the concept of trading when confronted with a subtraction problem at the beginning of the program. In English mathematics instruction, trading refers to grouping (or regrouping) of smaller (or bigger) units to bigger (or smaller) units such as grouping of 10 ones to 1 ten and regrouping of 1 ten to 10 ones. They made the following comments.

Participant 6: You think it's going to be easy to teach maths, and you've been given a practical example that you know is hard, and you have to sit there and work it through. And if you're doing that for the first time it's confusing and just thinking you can do [the algorithms] yourself really quick because we're all in the symbolic stage, but trying to communicate that back to how (i.e., conceptual knowledge) we would've originally learnt ourselves, and teaching it that way I think is hard.

Participant 3: It was difficult the first time I was practicing because it's been twenty-five years since I was in high school and to go back to primary school level, I was like 'I can't remember that'. So I found it quite difficult because obviously we're at the symbolic stage and that everybody here is capable and can do [the algorithms], so to revert back to the physical was a challenge.

The above responses indicate participants did not understand the reason primary students struggle with conceptualizing trading in algorithmic form. It was because typically they could readily calculate answers in their heads. Thus, they did not see the need to use other representations such as concrete objects and pictures to help students master a concept. After practising teaching for several weeks, they came to realize that developing a concept by growing cognitively from concrete level to symbolic level could effectively support students to build and to internalize an abstract concept. Participants 3 and 4 made the following notes which echoed Bruner's (1966) learning theory as discussed earlier in this paper:

> *Participant 4: I think for me it kind of put me in the right mind-set, like before when I came in, as [participant 1] said, I looked at the board and started figuring out how to do it, but now when I look at the board and look at the question I think 'okay, how am I going to best explain [the concept] that to the kids, using, do they need concrete, do they need pictorial or do they need symbolic?' It's getting into the minds of the students that you have and thinking where they're at and how you're going to best explain it so they can understand it.*

> *Participant 3: The concrete, pictorial and symbolic is very important depending on what learning stage the students are at, and even if they're in [key] stage 3, some children maybe just won't get it because they're a little bit behind everybody else, so it helps to just go back a step and reinforce things.*

Whilst the notion and concept of Bruner's three modes of representation were not new to the participants, they were not able to connect this theory to practice - they reported an inability to understand its application to real classroom teaching prior to the intervention program. The dilemma of the theory-practice nexus was reconciled after practising teaching for several weeks. The benefit of using the three modes of representation in teaching was grounded in the participants' practice, no longer just a theory. The change of the participants' belief in the need for the use of the three modes of representation for explaining a mathematical concept as reported in this study provided evidence that practice in teaching can be a means to approach the relationship between theory and real teaching. It helped develop the participants' awareness of the complexity of mathematics teaching such that they are more aware of the impact of their teaching on students' learning process.

5.1.2 Using manipulatives

Observation of early student presentations indicated the participants in this study were initially unaware of how concrete objects were used in instructional practice, and how to make connections between concepts and symbols via concrete objects. A participant made the following note.

> *Participant 3: Yeah, [using physical objects is] challenging and a bit frustrating, but when you're dealing with children I find I don't have a problem explaining things [without using any object].*

After they had been involved in the program for some time and had 'practised teaching' in a repetitive cycle for one semester, they reported a far more developed appreciation of the use of such concrete objects. The following notes demonstrate their growing understanding of the use of concrete objects in classroom teaching.

> *Participant 2: I was the same as [participant 3] with the concrete stage and that, based-ten blocks, modelling sort of thing, and trading, and you need to have that, children might need the concrete, the thing (i.e., based-ten blocks) in front of them to be able to work through the process. Some might not need it but you should start with that.*

> *Participant 4: I thought it was good basically because it kind of reinforced to me the concept of trading. I didn't actually get how to do that with blocks before. I knew how to do the algorithm in my head but in order to do that …… I found that probably the best thing for me was the blocks, because actually that was the thing that I used, I was able to start from that like 'can you represent me four hundred and three? Now we're going to take away one hundred and sixty-eight and we're going to take them away block by block.' And trade, so you actually use the blocks and that's what it's based on, not the algorithms. It was good.*

> *Participant 8: And if you're teaching children who aren't too sure you can always say to them 'look, there's the blocks if you need to go and use them' just have them there so that they can utilise that to, even if they just want to check their answers before they come to you or have it marked or whatever.*

At the beginning stage of practising teaching, the participants also experienced difficulties in using precise but simple language to explain abstract concepts to junior primary students who had not developed much mathematical language. After the program, the participants came to realise that by manipulating concrete objects, students could more readily visualize the concepts, thus students did not merely rely on teachers' verbal explanation for mastering a concept and in the mean time, learnt to master correct mathematical language gradually. Consequently, concrete objects made learning abstract concepts more readily accessible to early age students – that is, a child can do more abstract thinking if physical objects are given to manipulate – as reported earlier in the paper.

As reported earlier, the above interview episodes further illustrated the power of practice in teaching in enabling the participants to connect real world teaching with theory. They become more aware of the use of manipulatives in classroom teaching.

5.1.3 Sequencing a series of modelling tasks

Prior to this program, the participants did not recognise the need for teaching conceptual knowledge, and did not recognise how structured and sequenced modelling tasks could benefit students' acquisition of conceptual knowledge. They also interpreted learning as an outcome of teaching and did not conceive learning as a process of knowledge building. The participants made the following notes.

> *Participant 1: [Before attending this program], when you're thinking about teaching once we're finished I probably didn't think about a sequence. I would've thought 'okay we'll do all the subtractions altogether and wouldn't have thought about the* ***borrowing or carrying****, and would've thrown it all in together, more likely than… instead of building upon it I probably would've done more digits altogether and expected them to know.*

> *Participant 5: [Before attending the program], I also think the sequencing, they won't understand how to teach, like what to start with, what to move on to or how to actually interact with the students, like, how to model what they're* ***doing with the blocks****. Well they might but not know how to do it effectively.*

Following intervention, they admitted their naïve understanding of teaching and made the following notes.

> *Participant 4: I think with the sequencing stuff it was really good because normally in my lesson plans I understand where I want to start and I know where I want to go but I don't know how to get there. So this sequencing stuff is kind of teaching me 'okay, how do I get there* ***slowly building up*** *using scaffolding and starting with no trading, then trading, no trading, trading and then trading more and more. It was really good to be able to reach the goal, slowly but effectively.*

> *Participant 6: Also, how we can best use our questions to not only introduce our topic but to* ***scaffold [students'] learning****. So we can ask further questions that will build on what they know, like, introduce a concept, and then ask another question that introduces it a little more and then introduces it a little more, so you can basically build up their understanding.*

As mentioned earlier, a series of modelling tasks which embedded *variation* can foreground the critical features of a concept at different stages and therefore display these critical features gradually from particular to general. After the program, the participants realised that "borrowing and carrying" (i.e., trading) is one of the critical features in addition and subtraction of whole number. Hence, they came to recognise that the teaching of subtraction should be focused on demonstrating the concept of grouping and breaking up of 10 step-wise through manipulating with blocks. They also came to realise that construction of knowledge is a process in which teachers are required to make a good start and then scaffold students' thinking and learning in a way that knowledge is built progressively. The sequence of modelling tasks provided them a direction of how to articulate the learning process for their students. The following paragraph and the participants' notes will illustrate this point.

During the interview, the participants were also asked to recall the sequence of the modelling tasks which they repeatedly practised during the program. This interview question aimed to investigate the degree to which participants understood the sequence of subtraction tasks. The following episode demonstrates their understanding of the underlying features and structure of the subtraction tasks.

> *Participant 1: So the first one [it is* $59-23$*] you didn't have to trade, second one [it is* $54-17$*] you had to trade, third one [it is* $369-145$*] you didn't have to trade, fourth [it is* $365-137$*] you did, fifth one [it is* $325-167$*] you did have to trade and sixth one [it is* $305-167$*] you did have to trade.*

> *Participant 3: For scaffold[ing students' learning]. In terms of the fact that the first one you don't have to worry about trading anything. It's just nine takes away three, five takes away two and its nice and clean. The second one four minus seven you've got to trade. The third one because you've got three digits it probably looks more difficult so therefore you revert back to not having to trade. And then you just progressively get harder.*

> *Participant 9: I think that fifth [it is* $325-167$*] and sixth [it is* $305-167$*] you have to trade in to the tens column [as well] so you have to go across and then trade and then trade again.*

The participants were able to identify that the concept of trading was the object of learning to be discerned in learning subtraction and thus, was the invariant. They were also aware that the sequence of tasks which started with no trading in a two-digit number subtraction, followed by trading at the units, then no trading in a three-digit number subtraction, followed by trading at the units then tens, and lastly, trading at both units and tens was progressively increasing the difficulty and hence, trading at different place values was the variance. The participants' notes illustrate their understanding as well as confirming Watson and Mason's suggestion (2006) that well-structured modelling in tasks that display *variation* can help learners develop knowledge gradually.

The participants' notes provided evidence that through practising teaching, they have re-conceptualized the notions of learning, i.e., learning is no longer a transmission of knowledge but a process of knowledge building. They also have developed an understanding that teaching conceptual knowledge should be the focus of mathematics teaching. In addition, the practice enabled them to understand better the hierarchical structure of mathematical knowledge.

Overall, the practice enhanced the participants' understanding of the complexity of mathematics teaching especially in the aspect of the knowledge of content and teaching (Ball, Thames, & Phelps, 2008). The participants demonstrated their ability to identify critical features of a concept and analyze the choice of examples, models and materials for instructional purposes.

5.2 Discerning other 'critical aspects' of mathematics classroom instruction

This section will report the results of the second research aims - what other 'critical aspects' of mathematics classroom instruction (other than the three teaching skills as discussed in the last section) did participants identify after completing the intervention program. The participants were firstly asked to indicate what cognitive aspects of teaching mathematics that they felt they understood better than those pre-service teachers who did not participate in the program. The participants indicated that they came to think more about their students' learning, such as students' learning diversity and learning process. In the past, they had never thought about different solutions for a problem and would stick tightly to their default answers. Their perception of teaching was to pass on a particular 'method' for arriving at the default answer. After the program, they came to realise the *complexity* of students' learning abilities, styles and processes. Based on the idea that no two children are identical, participants 1 and 8 made the following notes:

Participant 8: I think we'll have a better understanding of how to cover all children and all abilities in our class, as opposed to you know just going 'oh this is what we're going to do', and just have the one set thing that we think is right. Now we've got a bit more of a practical sort of.

Participant 1: I think we have a better understanding of the different ways that children get to the right answer, you know it's not just all about how we do it for example the subtraction. There were different sorts of applications, but we both got the right answer. So you know that's a better knowledge of how kids go about things.

The participants were then asked to think about what features/events they would look for in determining an effective mathematics lesson. The followings are participants 2 and 9's responses:

Participant 9: Scaffolding. Individually looking at each student and trying to see where they're at. So working on the student's prior knowledge, looking at your students and saying 'are they getting this? If not what do we need to go back to?'

Participant 2: If they can do what you've taught, maybe extend them more to think differently like in different ways. So if they're able to do it don't just leave them there if you've got a bit more of the lesson to go, try and build them along a bit.

Participant 9: Maybe like, how to actually effectively present a problem and then work on scaffolding to actually build on student's knowledge of that.

The participants previously did not recognise that mathematics teaching required knowledge at the intersection of content and students (Ball, Thames & Phelps, 2008). They did not realize that understanding students' prior knowledge of, and misconceptions about, the topic being taught were incredibly important for providing quality, appropriate classroom instruction and scaffolding students' learning. Through practising teaching in this intervention program, they came to realize the *complexity* of connecting teaching content with students' existing knowledge system. They were also increasingly aware that continuingly assessing students' learning throughout a lesson gave direction to addressing students' learning needs and to adjusting the pace of teaching.

Prior to this program, the participants had been afraid of using mathematical terms in their practicum. After repetitive practice, they became aware of their own improper and at times ambiguous use of mathematical language. They also came to realize the need for using correct mathematical language and agreed that it should be one of the features for quality teaching. They made the following notes.

> *Participant 3: To reinforce [correct] terminology with the children when you're teaching. So when you introduce them to a new term and you explain it then keep using it and encourage them to use it.*

> *Participant 1: Explaining the mathematical terms and actually getting the students to replicate it, getting them to follow through, and understand what you're saying and explaining it how you would explain it to a class. I think has been a lot easier, using the correct terms too, instead of… I can't even remember the wrong terms at the moment!*

> *Participant 5: The use of terminology, see if it's correct or not. Instead of like, when it's trading, they say borrowing or stuff like that. Because I've actually practiced it over and over and, now I know the terminology so it should be easier. Before I was like, shy to do it, just because I thought I'd get it wrong so I was scared, and now it's just like, 'yeah it's cool".*

> *Participant 1: Ask a very exact question, right to the point, explicitly.*
>
> *Participant 4: And assessing whether or not what you're saying is effective or not, so whether or not you need to change that.*

They appreciated their improvement after several weeks' practice of teaching and realised the *importance* of using appropriate, clear and unambiguous language when giving instruction and explaining a concept.

Overall, after the intervention program, the participants' attention to the critical aspects of mathematics teaching, have been enhanced - namely the knowledge of content and teaching and the knowledge of content and students (Ball, Thames, & Phelps, 2008) such as students' learning process and prior knowledge, use of manipulatives, use of proper mathematical language, choosing and sequencing appropriately the modelling tasks and asking appropriate questions.. Their confidence and self-awareness in general have also been enhanced. The researchers would argue that it was because the "practice of teaching" program provided them with a safe environment in which they were free from a sense of failure and in which they felt able to expose themselves to critical and constructive feedbacks. The participants' responses provide evidence for Eilam and Poyas's suggestion (2006) that pre-service teachers' awareness of the *complexity* of teaching requires special training. This paper asserts that the intervention program, *Study of Teaching*, as introduced in this study, has provided this "special training" to the participants and successfully improved their abilities in identifying critical cognitive aspects of teaching.

6. Conclusion

This study investigated the effect of the intervention program, *Study of Teaching*, on pre-service teachers' acquisition of three designated mathematics teaching skills and how much they understood about the critical aspects of mathematics teaching, all of which are related to the understanding of the complexity of mathematics teaching. We began with the assumption that pre-service teachers' ability to understand the complexity of mathematics teaching requires special training - that is, understanding the complexity of mathematics instructional practice through "deliberate practice", as described in this paper. We proposed that the intervention program, *Study of Teaching*, as introduced in this study could engage pre-service teachers in "deliberate practice" and thus believe that, to a certain extent, this intervention program could enhance their abilities in discerning the critical aspects of mathematics teaching. This is because learning to teach should be considered as an active, constructive process in which pre-service teachers enact a newly taught task and, simultaneously be exposed to the complexity of teaching. We believed that *Study of Teaching* could provide a safe environment for pre-service teachers to "deliberately practise" their teaching.

Our findings clearly show that the pre-service teachers developed an increased awareness of the complexity of teaching in mathematics classroom. They became aware of the cognitive aspects of the classroom episodes and came to interpret cognitive events in the teaching process. They exhibited increasing focus on teaching conceptual knowledge through demonstrating the three modes of representation. They emphasized intellectual growth of mathematical knowledge from concrete level to symbolic level. They also came to value the use of manipulatives in teaching mathematics. They exhibited awareness of teaching activities with reference to their effect on students. For instance, they came to realise the role of correct mathematical language in scaffolding students' learning process. They also recognised that an effective process of knowledge building required well structured modelling tasks which displayed variation in the critical features of the discerned concepts. They highlighted students' learning diversity, prior knowledge and misconceptions for providing effective and quality teaching. Overall, they were able to connect the specifics of mathematics classroom interactions with broader principles of teaching and learning. Our findings have provided evidence that enactment of complex practice in settings of reduced complexity and with declared goals (i.e., practising three designated teaching skills) strengthened pre-service teachers' understanding of cognitive skills of teaching and brought forth their attention to noteworthy teaching events. Furthermore, our results have indicated that focussed microteaching in the setting, as in our intervention program, provided pre-service teachers with a safe environment to practise teaching. It was safe because pre-service teachers practised in an environment which, while free from a sense of failure and criticism, was rich in formative critique and feedback. In general, their awareness of the complexity and intricacy of actual classroom teaching of mathematics has been greatly improved following intervention.

Some limitations of the intervention and its study must be delineated. Firstly, the lack of a control group interview did not allow us to reinforce our conclusion with regards to the effects of the intervention program, *Study of Teaching*. We did not interview those pre-service teachers who chose not to participate in this program because, ethically, it was not right to ask them those questions from the interview guide which we believed that they did not have any (or sufficient) knowledge to respond to in an informed manner. It was not ethical, too, if they struggled with the interview questions and later on regretted that they had not participated in the program because they felt that they had missed something in their preparation as teachers. Secondly, although those pre-service teachers who participated in the program exhibited significant improvement, an extension of the program may enable them to reach a better understanding of the complexity of teaching that could impact their performance in practicum. Last, a follow-up study on those pre-service teachers who participated and did not participated in the program would enable us to examine their actual functioning in complex authentic mathematics classroom.

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